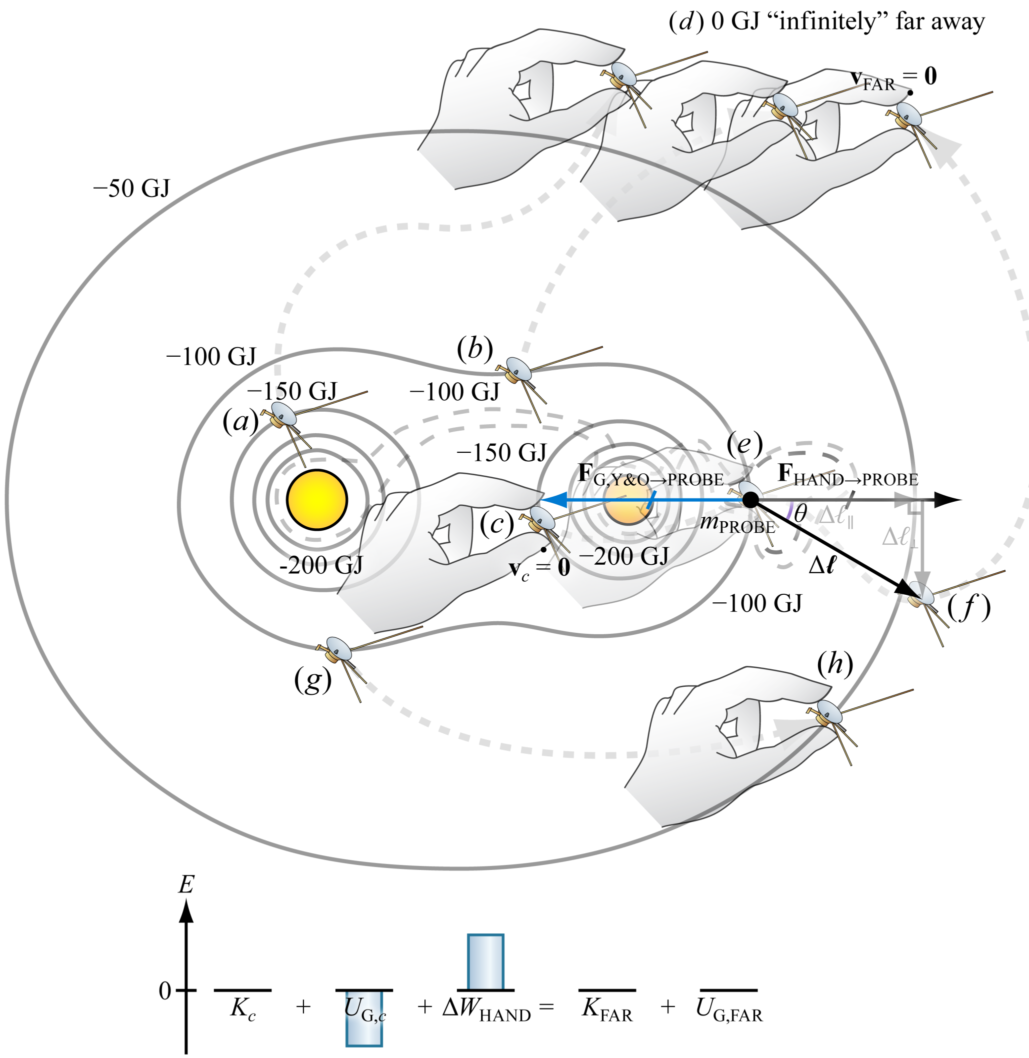
Potential energy landscapes (calculus-based physics)

Use differences in isoline labels to calculate amounts of work needed for transport between positions

$$-\Delta U_{\mathrm{F},1\ldots N} := \Delta W_{\mathrm{F},2\to 1} + \Delta W_{\mathrm{F},1\to 2} + \cdots + \Delta W_{\mathrm{F},N-1\to N}$$



Fix the position of each object except for one and make a map of potential energy as a function of the position of this single moveable object.

isoline – drawing of set of points on a map that share the same value of a variable of interest (e.g. potential energy isoline)

Consider a portion of a path along which the hand applies a force $\vec{\mathbf{F}}_{\mathrm{HAND}}$ that is locally anti/parallel to the displacement and basically balances out the force $\vec{\mathbf{F}}$ associated with the potential energy landscape.

$$|\Delta W_{\mathrm{HAND}}| = \left|\vec{\mathbf{F}}_{\mathrm{AVG}}\right| |\cos\theta| \Delta\ell$$

$$|\Delta U_{\mathrm{F}}| = \left|\vec{\mathbf{F}}_{\mathrm{AVG}}\right| \Delta\ell_{\parallel}$$

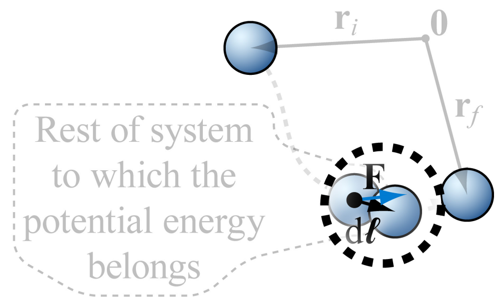
$$\left|\vec{\mathbf{F}}_{\mathrm{AVG}}\right| = \frac{|\Delta U_{\mathrm{F}}|}{\Delta\ell_{\parallel}}$$

($\vec{\mathbf{F}}$ points "downhill")

Potential energy landscapes (calculus-based physics)

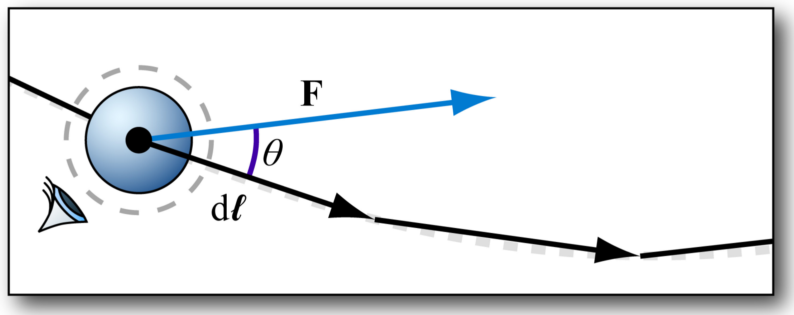
Using calculus to relate forces and potential energies

$$-\Delta U_{\mathrm{F}} := \Delta W_{\mathrm{F}}$$



$$\Delta U_{\mathrm{F},\vec{\mathbf{r}}_i\to\vec{\mathbf{r}}_f} = -\int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \vec{\mathbf{F}}\cdot \mathrm{d}\vec{\boldsymbol{\ell}}$$

$$\Delta U_{\mathrm{F},\vec{\mathbf{r}}_i\to\vec{\mathbf{r}}_f} = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} -\vec{\mathbf{F}}\cdot \mathrm{d}\vec{\boldsymbol{\ell}}$$



$$U_{\mathrm{F}}(\vec{\mathbf{r}}_f) - U_{\mathrm{F}}(\vec{\mathbf{r}}_i) = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} -\vec{\mathbf{F}}\cdot \mathrm{d}\vec{\boldsymbol{\ell}}$$

In the context of multivariable calculus, U_{F} is an "antiderivative" of $-\vec{\mathbf{F}}$, so $-\vec{\mathbf{F}}$ is the derivative of U_{F}.

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} + F_z\hat{\mathbf{k}} = -\frac{\partial U_{\mathrm{F}}}{\partial x}\hat{\mathbf{i}} - \frac{\partial U_{\mathrm{F}}}{\partial y}\hat{\mathbf{j}} - \frac{\partial U_{\mathrm{F}}}{\partial z}\hat{\mathbf{k}}$$

Potential energy landscapes (calculus-based physics)

Interpret graphs of potential energy functions

$$-\Delta U_{\mathrm{F}} := \Delta W_{\mathrm{F}}$$

For motion constrained to x-axis,

$$\Delta U_{\mathrm{F}} = -\int_{x=x_i}^{x=x_f} F_x \, \mathrm{d}x$$

The negative of the accrued signed area under the graph of the force function* provides the change in potential energy.

$$F_x = -\frac{\mathrm{d}U_{\mathrm{F}}}{\mathrm{d}x}$$

The negative of the slope of the potential energy function provides the force.

* If force cannot be expressed as a function of position alone, then no associated potential energy function exists.